Baroclinic Fluxes in the Gulf Stream Between 75° and 55°W

by Melinda M. Hall

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1. Introduction

The "baroclinic transport" T_{1000} of the Gulf Stream (defined by Hogg (1992) to be transport relative to 1000 dbar) has been examined at about 10 CTD and XBT sections between Cape Hatteras and the Grand Banks. Motivation for the work originated with Hogg's proposition that T_{1000} remains constant at 47 Sv over this distance, at least in the synoptic average sense. Furthermore, previous work on downstream change of Gulf Stream fluxes (Fofonoff and Hall, 1983) suggested looking at baroclinic momentum and kinetic energy fluxes (TM and TKE respectively) in addition to mass transport, and comparing their cross-stream structure with that predicted by an inertial jet model of the current.

Hogg (1992) used long time series of moored velocity and temperature measurements, and though average cross-stream structure is accessible with that data (by exploiting the strong relation between thermocline temperature and cross-stream position), instantaneous values of T_{1000} are not. In contrast, hydrographic data are useful for instantaneous assessments of the current, but may not represent the long-term average. In fact, it is found that T_{1000} diverges over 30% from the canonical value, with stronger variations in the associated momentum and kinetic energy fluxes. The latter are indicative of a dynamically (and energetically) active system, and being able to monitor them might offer insight into higher order Gulf Stream dynamics. A simple model is suggested for evaluating these baroclinic fluxes, so that a modest IES array could be used for monitoring them.

2. The Observations

In late March of 1988, a hydrographic survey of the Gulf Stream was carried out aboard *R/V Endeavor* (Fig. 1a), and included CTD (and XBT) sections at 68° and 55°W, as well as four additional XBT sections at roughly 66°, 64.5°, 63.5° and 59.5°W longitude. Hall and Fofonoff (1992) have discussed the two CTD sections in detail; here the focus is on the full suite of crossings. Three of the XBT sections consisted entirely of T5 drops, yielding full resolution of the current down to 1800 m at 66°, 64.5°, and 63.5°, while remaining sections included both T5's and T7's (to 760 m). A typical temperature secton (66°W) is shown in Fig. 1b for depths above 1600 m.

For the geostrophic velocity calculations, density was determined from the Armi and Bray (1982) T-S fit for the western North Atlantic, which is adequate for temperatures $T \leq 12^{o}C$. Above this level, temperature inversions lead directly to density inversions with this choice, so unstable density profiles were smoothed with a third-degree poynomial fit to data immediately above and below the inversion. The density equation can be inverted to obtain salinities for the fitted depths, if desired. Transport relative to any depth can be calculated from two drops bracketing the Gulf Stream's isopycnal drop, so even for sections where

both T5's and T7's were used it is possible to calculate T_{1000}. In contrast, TM and TKE depend heavily on the actual velocity structure; hence for sections where deep sampling was not uniform, fluxes have been calculated relative to 760 m. These cases are noted in the flux table.

Finally, 5 CTD sections near Cape Hatteras were kindly made available to me by Bob Pickart (Pickart and McKee, 1992), and these have been included for comparison. It should be noted that 1) some of these sections (intended to survey the Deep Western Boundary Current and not necessarily the Gulf Stream) do not cross the entire current; and 2) the station spacing is insufficient for calculating accurate momentum and kinetic energy fluxes, as discussed in the section on cross-stream resolution.

3. The Model

For the flux calculations, the thermocline Gulf Stream is modeled using a 1 1/2 layer (reduced baroclinic) inertial jet with constant potential vorticity (Fig. 2). However, the layer does not outcrop at the inshore edge of the current, but retains a constant value of h_c (for "cold"). Thus, the jet satisfies

$$u = \frac{-g'}{f}\frac{\partial h}{\partial y} \tag{1}$$

where $g' = -g\,\Delta\rho/\rho$, $\Delta\rho$ is the density jump across the interface, and

$$\frac{f - \partial u/\partial y}{h} = \frac{f}{h_w} \tag{2}$$

where h_w is the depth the interface attains on the warm side of the jet; in this work, h_w is fixed at 1000 m. Solving (1) and (2) with $h = h_c$ at $y = 0$ and $h \rightarrow h_w$ for $y \rightarrow -\infty$ yields an interface (for $y < 0$)

$$\begin{aligned} h &= h_w - \Delta h\, e^{y/R} \qquad (3)\\ \Delta h &= h_w - h_c \\ R &= \frac{(g'h_w)^{1/2}}{f} \end{aligned}$$

The expressions (1) and (3) may be used to evaluate mass, momentum and kinetic energy fluxes of the model jet; integrating southward from the core yields:

$$T(y) = \int_y^0 \rho u h\, dy = \frac{g'\Delta h}{f}\left[h_w - \frac{\Delta h}{2} - h_w e^{y/R} + \frac{\Delta h}{2}e^{2y/R}\right]$$

$$TM(y) = \int_y^0 \rho u^2 h\, dy = \frac{\rho g'^{3/2}}{f h_w^{1/2}}(\Delta h)^2\left[\frac{h_w}{6} + \frac{h_c}{3} - \frac{h_w}{2}e^{2y/R} + \frac{\Delta h}{3}e^{3y/R}\right]$$

$$TKE(y) = \int_y^0 \frac{1}{2}\rho u^3 h \; dy = \frac{\rho g'^2}{f \; h_w}(\Delta h)^3\left[\frac{h_w}{12} + \frac{h_c}{4} - \frac{h_w}{3}e^{3y/R} + \frac{\Delta h}{4}e^{4y/R}\right]$$

4. Fluxes

The mass, momentum and kinetic energy fluxes in the accompanying table are defined as follows:

$$T_{1000} = \rho \int_{y_s}^{y_n} dy \int_{1000m}^{sfc} dz \; u_{bc}$$

$$TM_{1000} = \rho \int_{y_s}^{y_n} dy \int_{1000m}^{sfc} dz \; u_{bc}^2$$

$$TKE_{1000} = \rho \int_{y_s}^{y_n} dy \int_{1000m}^{sfc} dz \; \frac{1}{2}u_{bc}^3$$

where y_s and y_n are the cross-stream limits of the strong velocity signature, and u_{bc} is the geostrophic velocity relative to 1000 m. The net model fluxes are obtained as $y \rightarrow -\infty$ in the expressions for $T(y)$, $TM(y)$, and $TKE(y)$:

$$T_{model} = \frac{\rho g'}{2f}(h_w^2 - h_c^2)$$

$$TM_{model} = \frac{\rho g'^{3/2}}{f h_w^{1/2}}(\Delta h)^2\left[\frac{1}{6}h_w + \frac{1}{3}h_c\right]$$

$$TKE_{model} = \frac{\rho g'^2}{2f \; h_w}(\Delta h)^3\left[\frac{1}{12}h_w + \frac{1}{4}h_c\right]$$

In these expressions, $h_w = 1000m$ (or 760 m as noted in tables), h_c is the depth of the delimiting isopycnal on the cold side of the stream, $\Delta h = h_w - h_c$, $g' = 10^{-2}m \; s^{-2}$, and f is the value of the coriolis parameter for each section (ranging from $(0.78 \rightarrow 0.94) \; x \; 10^{-4}s^{-1}$). Table 2 shows the values of σ_θ (the delimiting isopycnal), h_c, and Δh for the sections.

The Synoptic Average, or "Canonical" Stream

The synoptic average fluxes may be calculated from the model by assuming that the average drop across the Gulf Stream of the $12^o C$ isotherm -- about 600 m -- is representative of Δh. Then with h_w = 1000 m and $f = 0.9 \; x \; 10^{-4}s^{-1}$, we have h_c = 400 m, and

$$T_{1000} = 46.7 \; Sv$$

$$TM_{1000} = 37.9 \; x \; 10^9 \; N$$

$$TKE_{1000} = 22.0 \; x \; 10^9 J \; s^{-1}$$

for the "canonical" Gulf Stream. The transport value is that documented by Hogg (1992), but

estimates of the other fluxes are not presented in that work. In the sections of Table 1, only that at 66° W is close to the synoptic average.

5. Cross-Stream Resolution

Fofonoff and Hall (1983) discussed the effect of limited sampling on the evaluation of TM and TKE; mass transports, of course, can be calculated from just two stations bracketing the isopycnal drop across the current, as long as the velocity is in geostrophic balance. Because TM and TKE depend strongly on the the cross-stream structure of the velocity, the ability of the 1-1/2 layer model to predict these fluxes accurately suggests that it is a good representation of the baroclinic Gulf Stream. For the case described by Fofonoff and Hall, they showed that for station spacing on the order of the deformation radius, (TM, TKE) is underestimated by up to (20%, 40%) when the strong velocity core falls between stations, as it frequently does. Thus, typical CTD station spcing of 25 to 40 km is inadequate for calculating accurate momentum and kinetic energy fluxes for comparison with the model.

In contrast, all of the XBT sections of this study were characterized by resolution ranging from 8 km near the frontal structure (in some cases even less) to a maximum of 20 km in the rest of the current (the exception is the section at 55° W, compromised by severe weather conditions). Notice that CTD sections generally underpredict TM and TKE in Table 1. Figures 3a-c explicitly demonstrate the effect of limited sampling for the particular section at 66° W. Plotted are a) T(y), b) TM(y), and c) TKE(y), normalized by the net predicted by the model; the smooth curve is the model, while the others are the fully resolved XBT section and a decimated version of the same XBT section, with spacing of 20-35 km; they are distinguishable by the obviously lower resolution in the latter. Notice that not only are the total values of TM and TKE grossly underestimated when resolution is limited (in this case, by 17% and 30% respectively), but the cross-stream structure is distorted as well.

6. Conclusions

There appears to be a *time-varying* canonical baroclinic Gulf Stream, which may be defined (for convenience) relative to 1000 m. This canonical baroclinic structure has been modelled successfully by a 1-1/2 layer constant potential vorticity inertial model whose interface is defined by the isopycnal lying at 1000 m on the offshore side of the Stream (hence, its inshore depth may vary). In observations, adequate sampling of the current's structure is required to assess momentum and kinetic energy fluxes accurately (spacing of < 10 km in the core), as suggested previously by Fofonoff and Hall (1983). However, using the above model, one needs only the isopycnal drop across the current to determine these fluxes: this result suggests the ability to monitor time variability of the baroclinic fluxes with an IES array, for example, since this isopycnal drop roughly mirrors the drop in depth across the Gulf Stream of the 12° isotherm Z12, which can be determined from IES data. Deep XBT's (T5's) can be deployed to evaluate additional structure.

References

Armi, L. and N. A. Bray, 1982: A standard analytic curve of potential temperature versus salinity for the western North Atlantic. *J. Phys. Oceanogr., 12* (4), 384-387.

Fofonoff, N. P. and M. M. Hall, 1983: Estimates of mass, momentum and kinetic energy fluxes of the Gulf Stream. *J. Phys. Oceanogr., 13*, (10), 1868-1877.

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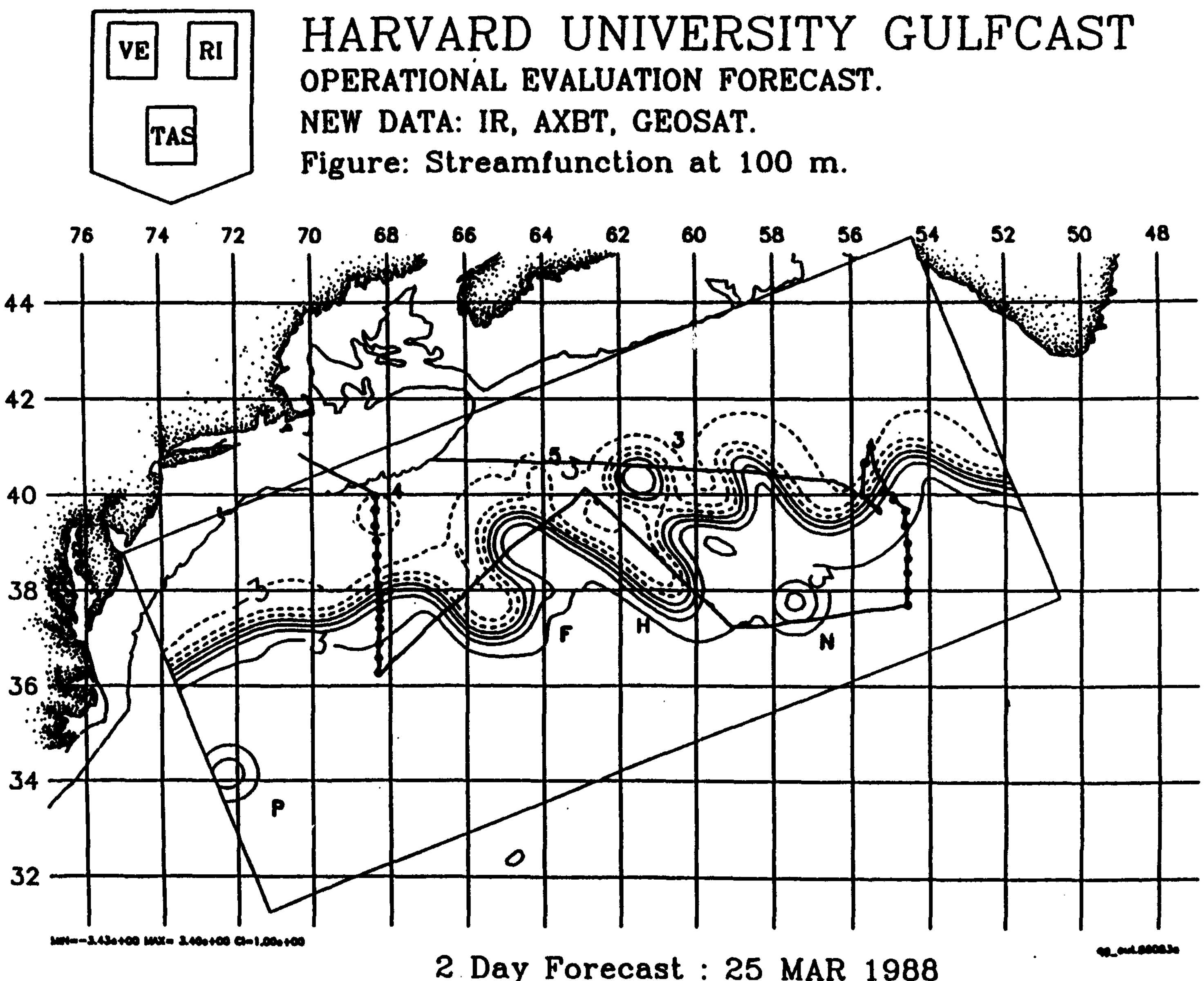
Hogg, N. G., 1991b: On the transport of the Gulf Stream between Cape Hatteras and the Grand Banks. *Deep-Sea Res., 39* (7/8), 1231-1246.

Mailing Address: Clark 3, W.H.O.I., Woods Hole, MA 02543

e-mail: mindy@latour.whoi.edu

Phone: (508) 457-2000, x2599

Fax: (508) 457-2181


VE
RI
TAS
HARVARD UNIVERSITY GULFCAST
OPERATIONAL EVALUATION FORECAST.
NEW DATA: IR, AXBT, GEOSAT.
Figure: Streamfunction at 100 m.
76
74
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2 Day Forecast : 25 MAR 1988

Figure 1a.

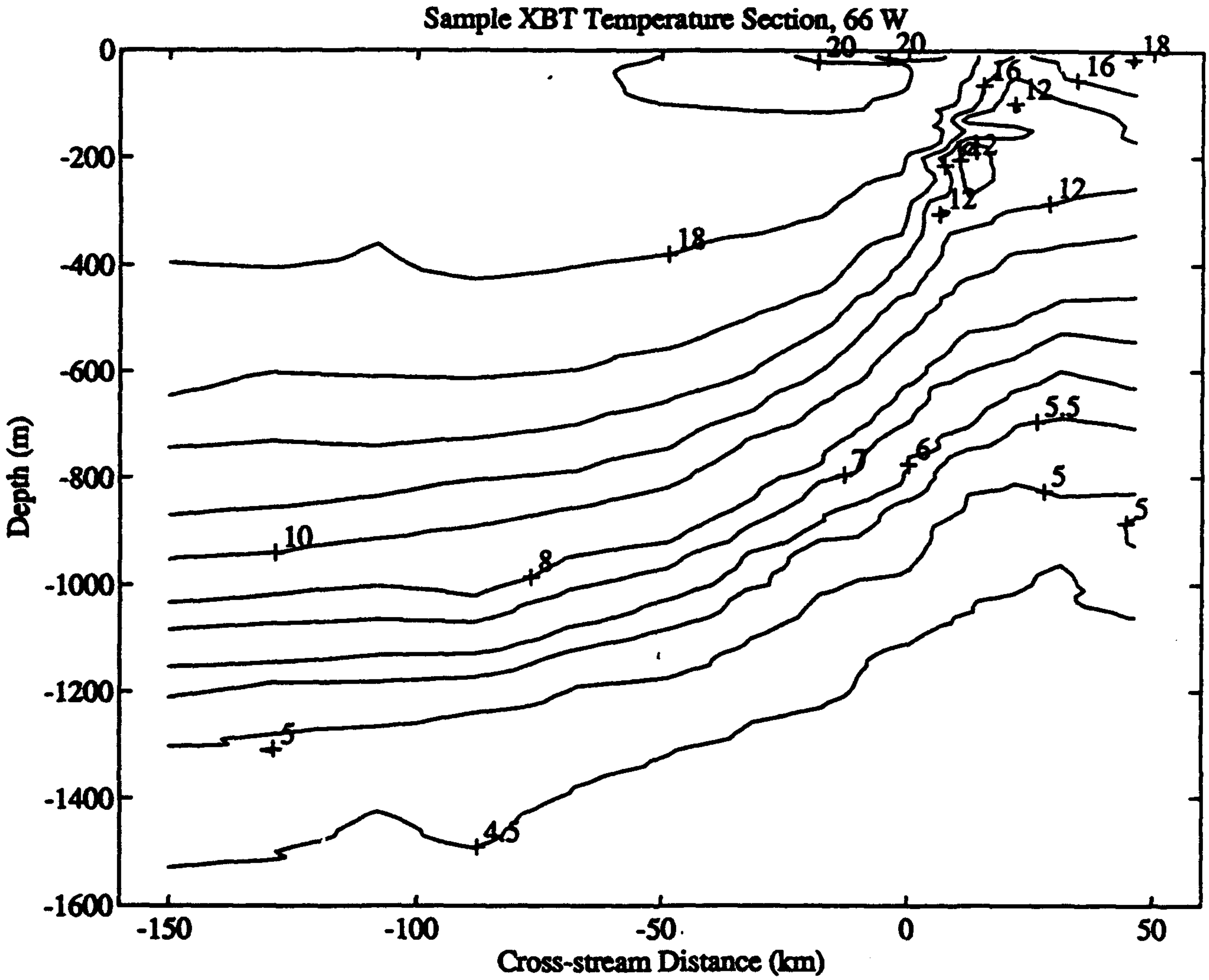
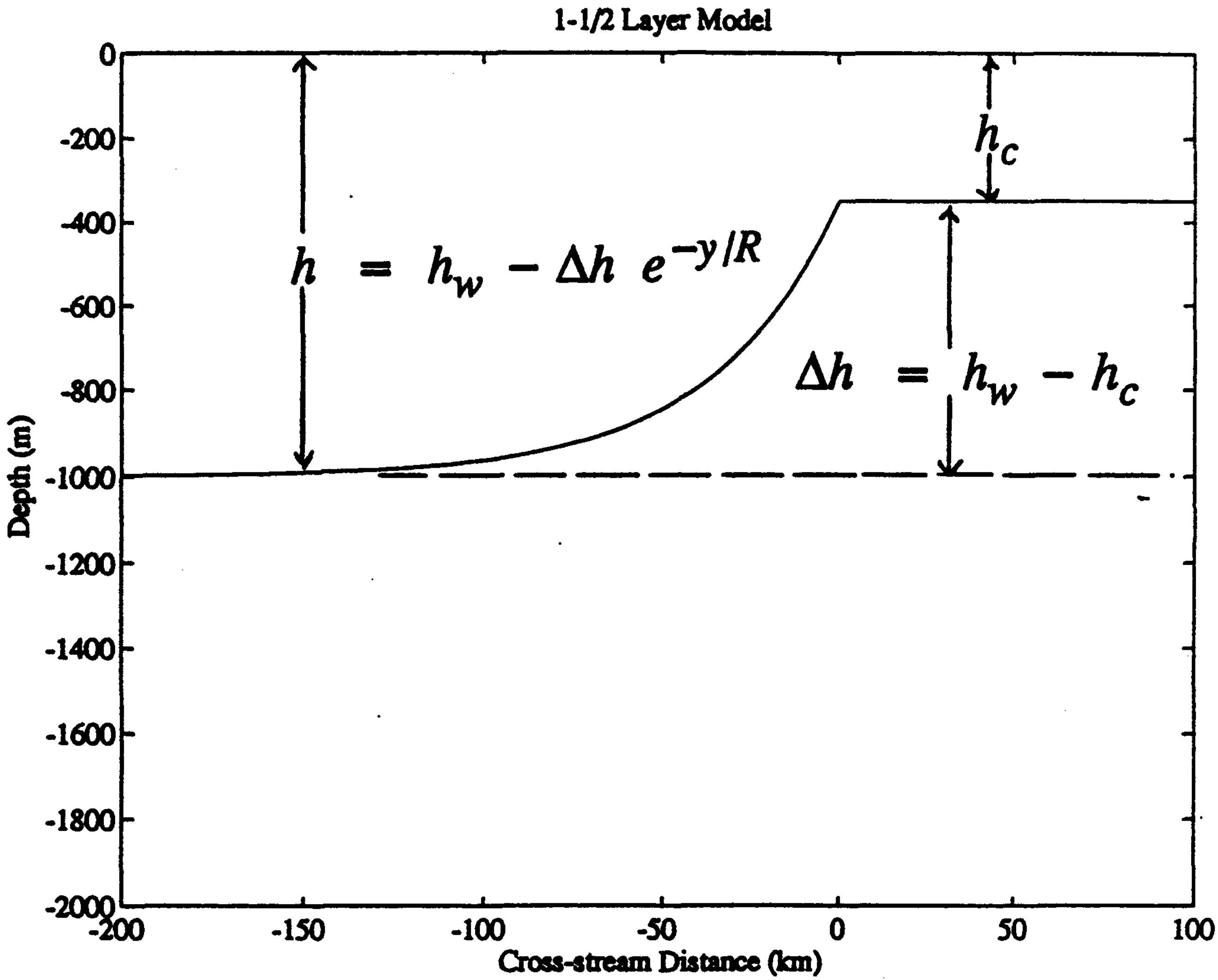

Sample XBT Temperature Section, 66 W
Depth (m)
Cross-stream Distance (km)
0
-200
-400
-600
-800
-1000
-1200
-1400
-1600
-150
-100
-50
0
50
20
20
18
16
16
12
12
12
12
18
5.5
7
6
5
5
10
8
5
4.5

Figure 1b.



1-1/2 Layer Model
$h = h_w - \Delta h\ e^{-y/R}$
h_c
$\Delta h = h_w - h_c$
Depth (m)
0
-200
-400
-600
-800
-1000
-1200
-1400
-1600
-1800
-2000
-200
-150
-100
-50
0
50
100
Cross-stream Distance (km)

Figure 2.

Location & Type		T_{1000} $(10^6$	T_{model} $m^3\ s^{-1})$	TM_{1000} $(10^9$	TM_{model} $N)$	TKE_{1000} $(10^9$	TKE_{model} $J\ s^{-1})$
76.5° W	CTD	47.5	43.9	37.8	27.9	17.4	7.9
74.0° W	CTD	57.9	53.7	40.5	48.3	18.5	31.3
72.5° W	CTD	53.0	51.8	38.2	45.6	17.0	29.3
71.0° W	CTD	56.6	52.9	42.1	49.5	20.8	33.6
68° W	CTD	53.5	51.3	33.0	46.8	14.2	31.0
68° W	XBT	55.7	52.0	28.7*	26.0*	18.5*	15.6*
66° W	XBT	44.8	46.0	35.0	36.5	21.7	20.5
64.5° W	XBT	32.5	36.3	22.8	21.9	11.3	9.2
63.5° W	XBT	30.4	36.3	18.6	22.4	8.0	9.7
		30.8	32.4	18.4	17.5	8.0	6.6
59.5° W	XBT	24.9*	26.7*	18.6*	16.1*	9.6*	6.9*
55.0° W	CTD	35.7	36.9	14.6	23.6	3.5	11.0
55.0° W	XBT	36.2	37.5	13.8	24.3	3.8	10.5

Table 1. Mass, momentum and kinetic energy fluxes of the Gulf Stream at 10 locations, relative to 1000 m except for asterisked values, which are relative to 760 m. Both observed and model-predicted values are given. At 63.5° W, results are given for 2 different choices of the bracketing XBT drops. Section at 76.5° W intersects the bottom on inshore side.

Section Location & Type	σ_θ (h_w = 1000 m) (kg m^{-3})	h_c (σ_θ) (m)	Δ h (m)
76.5° W CTD	27.491	557	443
74.0° W CTD	27.255	311	689
72.5° W CTD	27.398	325	675
71.0° W CTD	27.308	269	731
68° W CTD	27.274	294	706
68° W XBT	27.249	270	730
	26.806*	0*	760*
66° W XBT	27.348	420	580
64.5° W XBT	27.589	580	420
63.5° W XBT	27.589	570	430
	27.589	630	370
59.5° W XBT	27.189*	320*	440*
55° W CTD	27.462	554	446
55° W XBT	27.433	545	455

Table 2. Potential density σ_θ at 1000 m on the "warm" side of the current; the depth h_c of σ_θ on the "cold" side; and the difference $\Delta h = h_w - h_c$, which appears in the expressions for the model fluxes.

* Asterisked values are relative to 760 m.

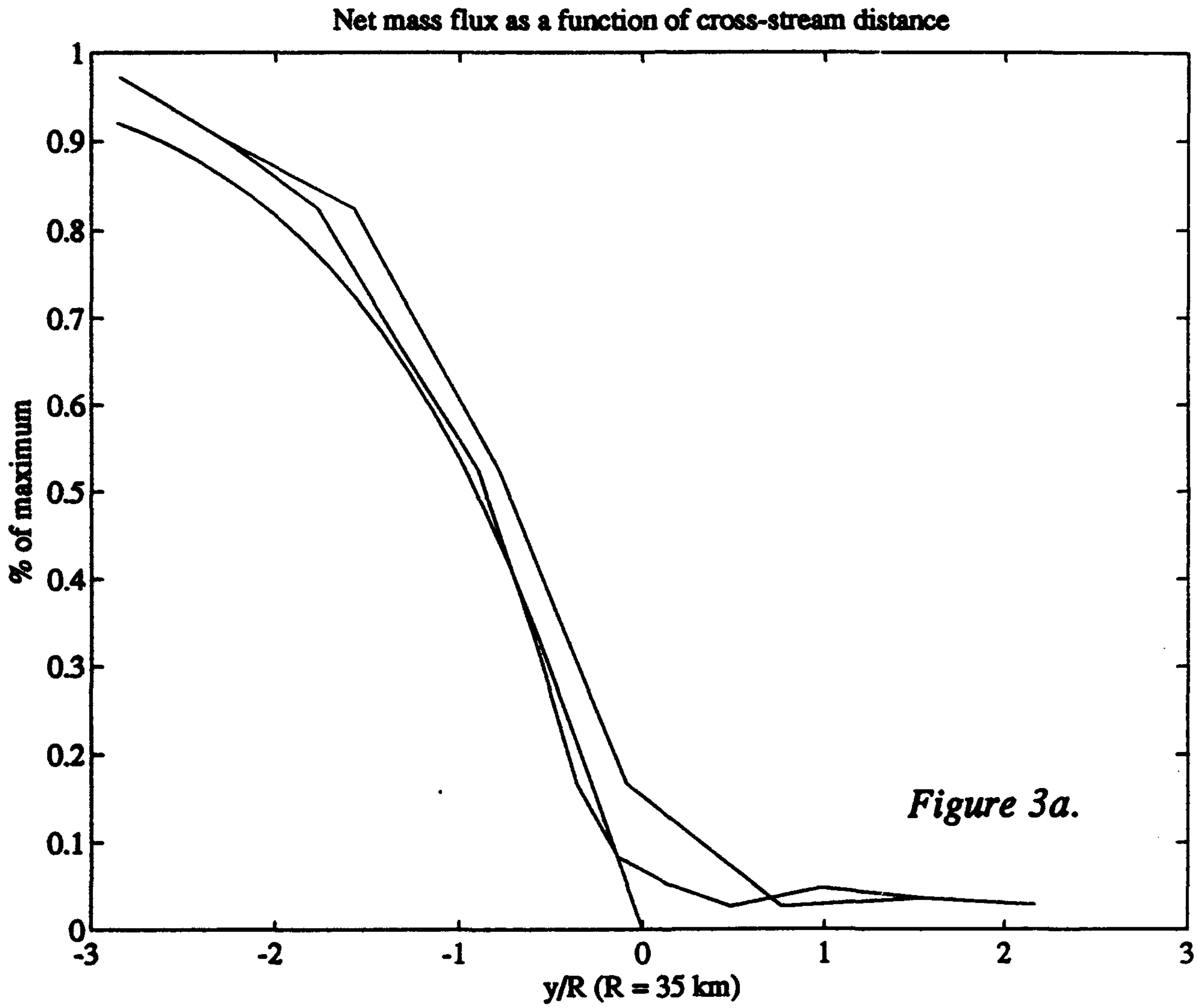

Net mass flux as a function of cross-stream distance
% of maximum
y/R (R = 35 km)
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-3
-2
-1
0
1
2
3

Figure 3a.

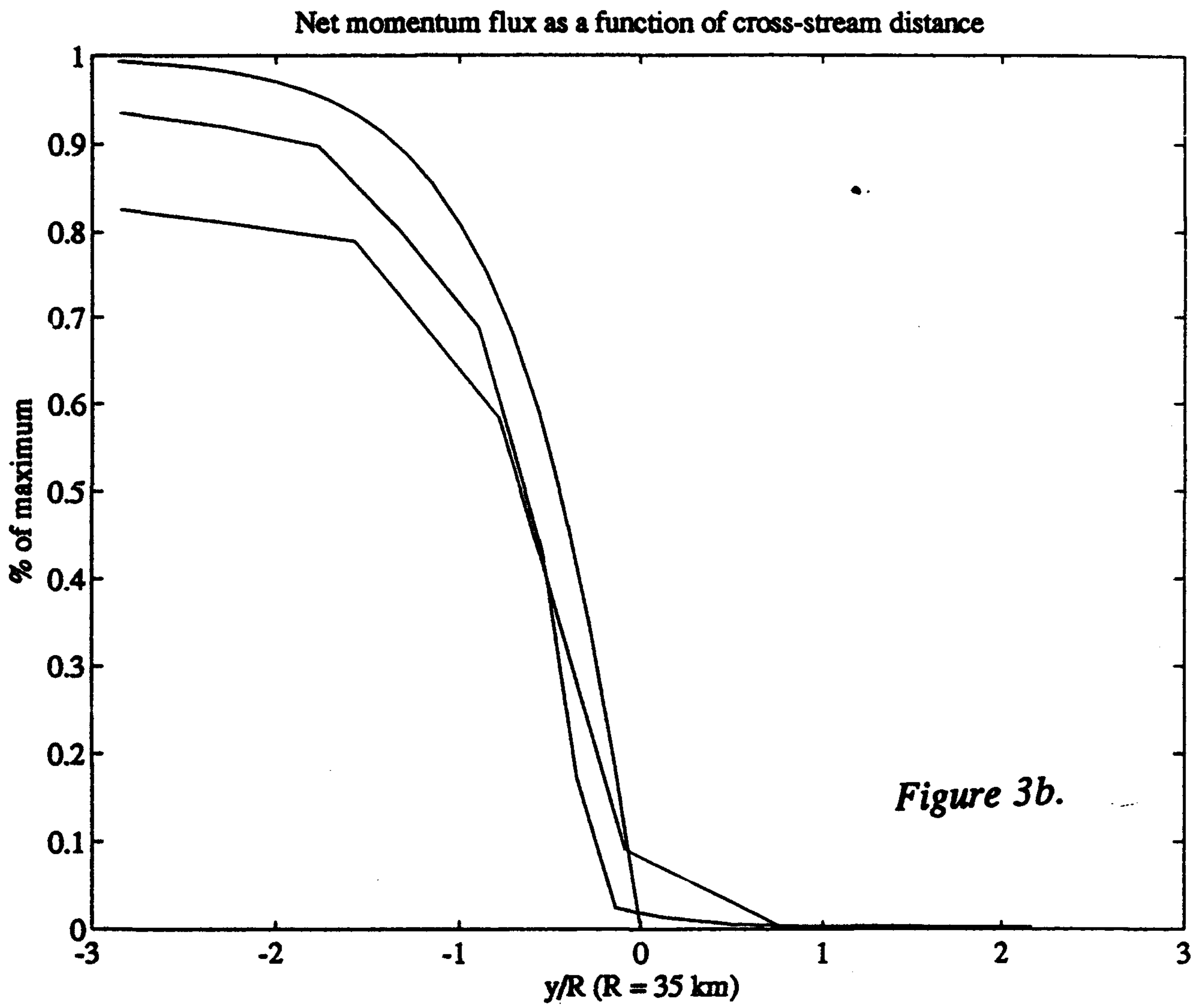

Net momentum flux as a function of cross-stream distance
% of maximum
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
-3
-2
-1
0
1
2
3
y/R (R = 35 km)

Figure 3b.

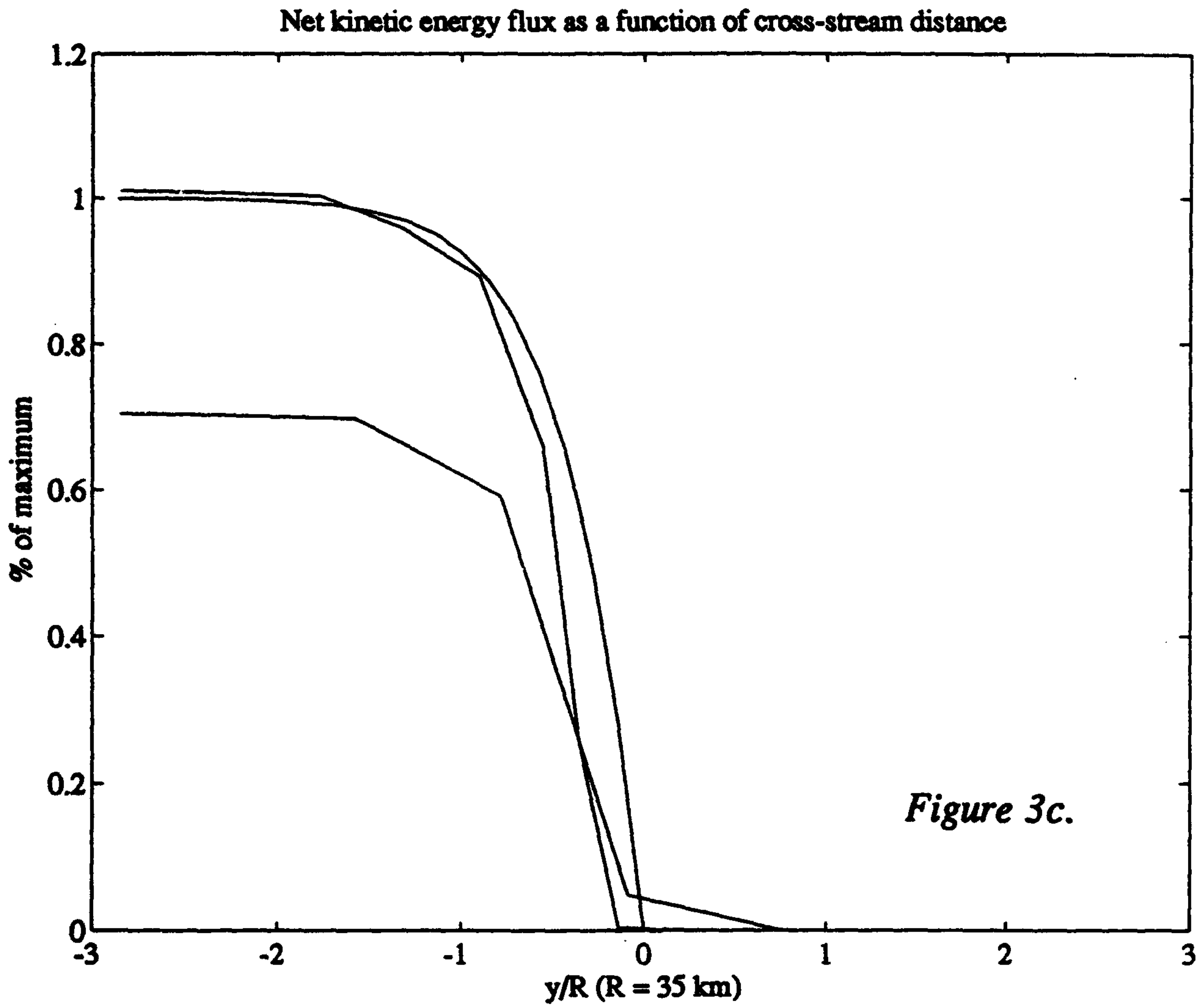

Net kinetic energy flux as a function of cross-stream distance
1.2
1
0.8
0.6
0.4
0.2
0
% of maximum
-3
-2
-1
0
1
2
3
y/R (R = 35 km)

Figure 3c.